① $\displaystyle\int x^3 e^{2x}\,dx =$

$+$	x^3	e^{2x}
$-$	$3x^2$	$\frac{1}{2}e^{2x}$
$+$	$6x$	$\frac{1}{4}e^{2x}$
$-$	6	$\frac{1}{8}e^{2x}$
$+$	0	$\frac{1}{16}e^{2x}$

$$\boxed{\frac{1}{2}x^3e^{2x} - \frac{3}{4}x^2e^{2x} + \frac{3}{4}xe^{2x} - \frac{3}{8}e^{2x} + C}$$

② $\displaystyle\int \frac{8x+17}{x^2+3x-4}\,dx = \int \frac{8x+17}{(x+4)(x-1)}\,dx$

$$\frac{A}{x+4} + \frac{B}{x-1} = \frac{8x+17}{(x+4)(x-1)}$$

$$Ax - A + Bx + 4B = 8x + 17$$

$$A + B = 8$$
$$-A + 4B = 17$$

$$-A + 4(8-A) = 17$$
$$-A + 32 - 4A = 17$$
$$-5A = -15$$
$$A = 3$$
$$B = 5$$

$$\int \left(\frac{3}{x+4} + \frac{5}{x-1}\right) dx$$

$$\boxed{3\ln|x+4| + 5\ln|x-1| + C}$$

(3) $$\int \frac{x^2+7x+4}{x(x+2)^2}\,dx$$

$$\frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2} = \frac{x^2+7x+4}{x(x+2)^2}$$

$$A(x+2)^2 + Bx(x+2) + Cx = x^2+7x+4$$

for $x=-2$:

$$-2C = -6$$
$$C = 3$$

for $x=0$:

$$4A = 4$$
$$A = 1$$

for $x=1$, $A=1$ and $C=3$:

$$9 + 3B + 3 = 12$$
$$3B = 0$$
$$B = 0$$

$$\int \left(\frac{1}{x} + \frac{3}{(x+2)^2} \right) dx$$

$$\boxed{\ln|x| - \frac{3}{x+2} + C}$$

(4) $\int x^2 \ln(x)\, dx$

$u = \ln(x)$ $\quad dv = x^2 dx$

$du = \frac{1}{x} dx$ $\quad V = \frac{1}{3}x^3$

$\frac{1}{3}x^3 \ln(x) - \int \frac{1}{3}x^3 \frac{1}{x}\, dx$

$\frac{1}{3}x^3 \ln(x) - \frac{1}{3}\int x^2\, dx$

$$\boxed{\frac{1}{3}x^3 \ln(x) - \frac{1}{9}x^3 + c}$$

(5) $\lim_{x \to 1} \frac{\sqrt{x+8} - 3}{5x^2 - 5} = \frac{0}{0}$

$\lim_{x \to 1} \frac{\frac{1}{2}(x+8)^{-\frac{1}{2}}}{10x} = \boxed{\frac{1}{60}}$

⑥ $\int e^{2x}\cos(3x)\,dx$ $\quad u = \cos(3x) \quad dv = e^{2x}\,dx$

$\quad du = -3\sin(3x) \quad v = \frac{1}{2}e^{2x}$

$$\frac{1}{2}e^{2x}\cos(3x) - \int \left(\frac{1}{2}e^{2x}\right)\left(-3\sin(3x)\right)dx$$

$u = -3\sin(3x) \quad dv = \frac{1}{2}e^{2x}\,dx$

$du = -9\cos(3x) \quad v = \frac{1}{4}e^{2x}$

$$\int e^{2x}\cos(3x)\,dx = \frac{1}{2}e^{2x}\cos(3x) - \left[-\frac{3}{4}e^{2x}\sin(3x) - \int -\frac{9}{4}e^{2x}\cos(3x)\,dx\right]$$

$$\int e^{2x}\cos(3x)\,dx = \frac{1}{2}e^{2x}\cos(3x) + \frac{3}{4}e^{2x}\sin(3x) - \frac{9}{4}\int e^{2x}\cos(3x)\,dx$$

$+\frac{9}{4}\int e^{2x}\cos(3x)\,dx \qquad\qquad +\frac{9}{4}\int e^{2x}\cos(3x)\,dx$

$$\frac{13}{4}\int e^{2x}\cos(3x)\,dx = \frac{1}{2}e^{2x}\cos(3x) + \frac{3}{4}e^{2x}\sin(3x)$$

$$\int e^{2x}\cos(3x)\,dx = \frac{4}{13}\left[\frac{1}{2}e^{2x}\cos(3x) + \frac{3}{4}e^{2x}\sin(3x)\right] + C$$

(7) $$\int \frac{5x^2-5x+15}{x\,(x^2+5)}\,dx$$

$$\frac{A}{x} + \frac{Bx+C}{x^2+5} = \frac{5x^2-5x+15}{x(x^2+5)}$$

$$Ax^2 + 5A + Bx^2 + Cx = 5x^2 - 5x + 15$$

For $x=0$:

$5A = 15$

$A = 3$

$A + B = 5$

$3 + B = 5$

$B = 2$

$C = -5$

$$\int \left(\frac{3}{x} + \frac{2x-5}{x^2+5}\right)dx = \int \frac{3}{x} + \frac{2x}{x^2+5} - \frac{5}{x^2+5}$$

$$\boxed{= 3\ln|x| + \ln(x^2+5) - \frac{5}{\sqrt{5}}\arctan\left(\frac{x}{\sqrt{5}}\right) + C}$$

(8) $\int x^4 \sin(2x)\, dx$

$+$	x^4	$\sin(2x)$
$-$	$4x^3$	$-\frac{1}{2}\cos(2x)$
$+$	$12x^2$	$-\frac{1}{4}\sin(2x)$
$-$	$24x$	$\frac{1}{8}\cos(2x)$
$+$	24	$\frac{1}{16}\sin(2x)$
$-$	0	$-\frac{1}{32}\cos(2x)$

$$\boxed{\frac{1}{2}x^4\cos(2x) + x^3\sin(2x) + \frac{3}{2}x^2\cos(2x) - \frac{3}{2}x\sin(2x) - \frac{4}{3}\cos(2x) + c}$$

(9) $\int \frac{3x^3 + 9x + 2}{(x^2+3)^2}\, dx$: $\frac{Ax+B}{x^2+3} + \frac{Cx+D}{(x^2+3)^2} = \frac{3x^3+9x+2}{(x^2+3)^2}$

$(Ax+B)(x^2+3) + Cx + D = 3x^3 + 9x + 2$

$Ax^3 + 3Ax + Bx^2 + 3B + Cx + D = 3x^3 + 9x + 2$

$A = 3$ $\quad B = 0$ $\quad 3A + C = 9$ $\quad 3B + D = 12$

$3(3) + C = 9 \quad D = 12$

$C = 0$

$$\int \left(\frac{3x}{x^2+3} + \frac{12}{(x^2+3)^2}\right)dx$$

$$\boxed{\frac{3}{2}\ln(x^2+3) + C + 12\int \frac{1}{(x^2+3)^2}\, dx}$$

(10) $$\int e^{2x} \sin(4x)\,dx$$

$u = \sin(4x)$

$du = 4\cos(4x)\,dx$

$dv = e^{2x}\,dx$

$v = \frac{1}{2}e^{2x}$

$$\frac{1}{2}e^{2x}\sin(4x) - \int \frac{1}{2}e^{2x}(4\cos(4x))\,dx$$

$u = 4\cos(4x) \qquad dv = \frac{1}{2}e^{2x}\,dx$

$du = -16\sin(4x) \qquad v = \frac{1}{4}e^{2x}$

$$\int e^{2x}\sin(4x)\,dx = \frac{1}{2}e^{2x}\sin(4x) - \left[e^{2x}\cos(4x) - \int -4e^{2x}\sin(4x)\,dx\right]$$

$$\int e^{2x}\sin(4x)\,dx = \frac{1}{2}e^{2x}\sin(4x) - e^{2x}\cos(4x) - 4\int e^{2x}\sin(4x)\,dx$$

$$+4\int e^{2x}\sin(4x)\,dx \qquad\qquad +4\int e^{2x}\sin(4x)\,dx$$

$$5\int e^{2x}\sin(4x)\,dx = \frac{1}{2}e^{2x}\sin(4x) - e^{2x}\cos(4x)$$

$$\int e^{2x}\sin(4x)\,dx = \frac{1}{5}\left[\frac{1}{2}e^{2x}\sin(4x) - e^{2x}\cos(4x)\right] + C$$

(11) $$\lim_{x \to \infty} \frac{x^3}{e^x} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{3x^2}{e^x} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{6x}{e^x} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{6}{e^x} = \boxed{0}$$

(12) $$\lim_{x \to 0} \frac{e^x - (1 - x)}{x} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{e^x + 1}{1} = \boxed{2}$$